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$$B=-\frac{8+4bD+4cE+bcDE}{2bc}.$$

The co-ordinates of the center of the ellipse are $-\dfrac{2D}{DE+2B}$ and $-\dfrac{2E}{DE+2B}$, or substituting the value of $B$, $\dfrac{Dbc}{2(2+bD+cE)}$ and $\dfrac{Ebc}{2(2+bD+cE)}$. The co-ordinates of the two Brocard Points $O$ and $O'$ are, respectively,

$$\frac{a^2b^2c}{a^2b^2+a^2c^2+b^2c^2},\ \frac{b^3c}{a^2b^2+a^2c^2+b^2c^2},\ \text{and}\ \frac{b^2c^3}{a^2b^2+a^2c^2+b^2c^2},\ \frac{a^2c^2b}{a^2b^2+a^2c^2+b^2c^2}.$$

Therefore, the co-ordinates of the middle point of $OO'$, that is, the center of the ellipse, are

$$\tfrac{1}{2}.\frac{b^2c(a^2+c^2)}{a^2b^2+a^2c^2+b^2c^2}\ \text{and}\ \tfrac{1}{2}.\frac{bc^2(a^2+b^2)}{a^2b^2+a^2c^2+b^2c^2};$$

$$\therefore \frac{D}{2+bD+cE}=\frac{b(a^2+c^2)}{a^2b^2+a^2c^2+b^2c^2},\quad \frac{E}{2+bD+cE}=\frac{c(a^2+b^2)}{a^2b^2+a^2c^2+b^2c^2};$$

whence, $D=-\dfrac{2\,(a^2+c^2)}{bc^2}$, $E=-\dfrac{2\,(a^2+b^2)}{b^2c}$.

Substituting, we find for the required equation of the Brocard Ellipse,

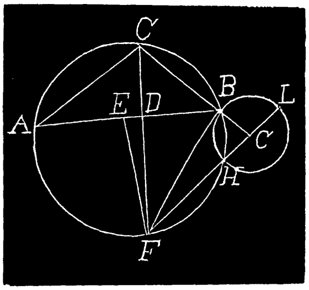
$$b^2(a^2+c^2)^2y^2+2bc[(a^2b^2+a^2c^2+b^2c^2)-a^4]xy+c^2(a^2+b^2)^2x^2$$
$$-2b^3c^2(a^2+c^2)y-2b^2c^3(a^2+b^2)x+b^4c^4=0.$$

**Also solved by G. B. M. Zerr, who used trilinear co-ordinates.**

**310. Proposed by L. H. MacDONALD, A. M., Ph. D., Sometime Tutor in the University of Cambridge, Jersey City, N. J.**

Construct a plane triangle having given the base, the vertical angle, and the bisector of the vertical angle.

**Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.; J. SCHEFFER, A. M., Hagerstown, Md., and C. N. SCHMALL, A. B., 89 Columbia Street, New York.**



Upon the given base $AB$ construct a circle whose segment $ACB$ shall contain the given vertical angle. Through $E$, the mid-point of $AB$, draw $EF$ perpendicular to $AB$, meeting the circumference at $F$. Join $FB$, and perpendicular to $FB$ draw $BG$ equal to one half the given bisector of the vertical angle. With $G$ as center and $BG$ as radius describe the circle $BHL$, and draw $FGL$. With $F$ as center, $FL$ as radius, describe a circle cutting the given circle in $C$. Join $FC$, cutting $AB$ in $D$. Then $ABC$ is the triangle required.

In the triangles *FCB* and *FBD*, $\angle FCB=\angle FBA$, since arc $AF=$arc $FB$; also $\angle CFB$ is common, hence the triangles are similar, and $FC:FB=FB:FD$; but $FL(=FC):FB=FB:FH$. Therefore $FH=FD$ and $HL=CD$.

Hence in the triangle *ABC*, *AB* is the given base, $\angle ACB$ the given vertical angle, and *CD* the given bisector, and the triangle is satisfied in every condition.

Also solved by **L. E. Newcomb**, and **A. H. Holmes**.

## CALCULUS.

**233. Proposed by W. J. GREENSTREET, M. A., Editor of the Mathematical Gazette, Stroud, England.**

Prove that $\int_0^\infty \frac{x^{a-1}dx}{1+2x\cos\theta+x^2}=\frac{\pi\sin(1-a)\theta}{\sin a\pi\,\sin\theta}$.

**Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.**

$$\int_0^\infty \frac{a^{-1}dx}{1+2x\cos\theta+x^2}=\frac{1}{a}\int_0^\infty \frac{dx}{\sin^2\theta+(x+\cos\theta)^2}$$

$$=\frac{1}{a\sin\theta}\tan^{-1}\left(\frac{x+\cos\theta}{\sin\theta}\right)_0^\infty=\frac{\theta}{a\sin\theta}$$

The problem giving the result stated is as follows:

$$\int_0^\infty \frac{x^{a-1}dx}{1+2x\cos\theta+x^2}=\frac{1-\sqrt{(-1)}\cot\theta}{2}\int_0^\infty \frac{x^{a-2}dx}{x+\cos\theta+\sqrt{(-1)}\sin\theta}$$

$$+\frac{1+\sqrt{(-1)}\cot\theta}{2}\int_0^\infty \frac{x^{a-2}dx}{x+\cos\theta-\sqrt{(-1)}\sin\theta}=P.$$

Let $x=y[\cos\theta\pm\sqrt{(-1)}\sin\theta]$, the plus sign for the first term, the minus sign for the second term.

$$\therefore P=\tfrac{1}{2}[1-\sqrt{(-1)}\cot\theta]\,[\cos(a-2)\theta+\sqrt{(--1)}\sin(a-2)\theta]\int_0^\infty \frac{y^{a-2}dy}{1+y}$$

$$+\tfrac{1}{2}[1+\sqrt{(-1)}\cot\theta][\cos(a-2)\theta-\sqrt{(-1)}\sin(a-2)\theta]\int_0^\infty \frac{y^{a-2}dy}{1+y}.$$

$$\therefore P=[\cos(a-2)\theta+\sin(a-2)\theta\cot]\int_0^\infty \frac{y^{a-2}dy}{1+y}$$

$$=\frac{\sin(a-1)\theta}{\sin\theta}\int_0^\infty \frac{y^{a-2}dy}{1+y}=\frac{\sin(a-1)\theta}{\sin\theta}\cdot\frac{\pi}{\sin(a-1)\pi}.$$

$$\therefore P=\frac{\pi\sin(1-a)\theta}{\sin\theta\,\sin a\pi}$$

This problem was incorrectly stated, the error being due to an oversight in reading proof. It is correctly stated above, the numerator being $x^{a-1}$ instead of $a^{-1}$. ED. F.